A COMPUTER PROGRAM FOR THE VON HIPPEL METHOD OF THE MEASUREMENT OF DIELECTRIC CONSTANT

Open File Report 82-20

ISSN 0712-4562

NATURAL RESOURCES
NEW BRUNSWICK
RESSOURCES NATURELLES
NOUVEAU-BRUNSWICK

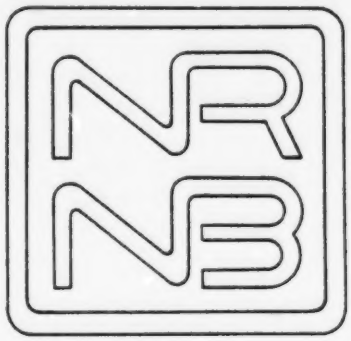
This report has been reprinted by:

Mineral Resources Division
Department of Natural Resources
Province of New Brunswick

Hon. E. G. Allen, Minister

1982

A COMPUTER PROGRAM FOR THE VON HIPPEL METHOD OF THE MEASUREMENT OF DIELECTRIC CONSTANT



Mineral Resources Division
Department of Natural Resources
New Brunswick

Copies available by contacting:

Mineral Resources Division
Department of Natural Resources
P. O. Box 6000, Fredericton, N.B.
E3B 5H1

Report No.: Open File Report 82-20

Author: Segundo J. Vasquez, University of New Brunswick, Dept. of Electrical Engineering

Cover: Tim Fevens

Catalogue No.: ISSN 0712-4562

A COMPUTER PROGRAM FOR THE VON HIPPEL METHOD OF THE MEASUREMENT OF DIELECTRIC CONSTANT

by

Segundo J. Vasquez

Submitted in partial fulfillment
of the Requirements for the Degree of
Bachelor of Science in Electrical Engineering
at the University of New Brunswick

December 1981

ABSTRACT

A computer program was developed for the calculation of dielectric properties of oil shale materials from measurements obtained on short-circuited coaxial lines or rectangular waveguides.

The measurement of these dielectric properties (ε_r' and $\tan\delta$) is based on Von Hippel's method. The sample of the material whose ε_r' and $\tan\delta$ are to be measured is placed in a shorted section of a waveguide and the resulting standing-wave patterns are probed with a slotted section. The sample has the form of rectangular blocks having the same cross section as that of a standard X-band or S-band waveguide; or in the form of powder in the case of coaxial lines.

One of the advantages of this method is that it allows measurements to be made very quickly, using small amounts of materials.

Graphical illustration and necessary mathematical relationships are presented. A copy of the program is included along with examples of input and output data.

ACKNOWLEDGEMENT

The author would like to express his sincere appreciation to Dr. J.E. Lewis for suggesting the topic of this thesis, as well as providing invaluable assistance throughout the entire project. My sincere thanks to Dr. J. Tranquilla and graduate student Bill Briggs for their throughtful questions and for being my second readers.

Above all, the author wishes to thank his wife, Maria, and daughter, Angelica, for their unmeasured patience during the study.

TABLE OF CONTENTS

		Page
Chapter 1	INTRODUCTION	1
Chapter 2	VON HIPPEL METHOD	3
2.1	Measurement techniques	3
2.2	Equations describing the method	5
2.3	Approximated solution to $\frac{\tanh\gamma_2 d}{\gamma_2 d} = Ce^{j\xi}$	7
Chapter 3	COMPUTER PROGRAM DESCRIPTION	8
3.1	Subprogram DIELEC	9
3.2	Subroutines used by DIELEC	11
3.3	Flowchart	13
Chapter 4	CONCLUSIONS AND RESULTS	15
REFERENCES		16
APPENDIX 1 -	PROGRAM LISTING	17
APPENDIX II-	MATHEMATICAL THEORY	23
APPENDIX III -	SAMPLE OUTPUT	29

CHAPTER 1

INTRODUCTION

Knowledge of the dielectric properties of materials and their frequency and temperature dependence are very important in many applications. In the study for which this computer program was developed, data was needed in a broad range of frequencies on the dielectric properties of oil shale materials to identify the most promising frequency ranges for possible pyrolysis by selective dielectric heating. The program and method of measurements described here, however, is applicable to a wide variety of materials.

The dielectric properties of interest in this thesis are the relative permittivity or dielectric constant and the loss tangent of the dielectric. The complex permittivity is given by

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r'(1 - j\tan\delta) \qquad [1]$$

where

ε_r' = The dielectric constant (the dielectric constant influences the amount of energy that can be stored in the form of electric fields[2]).

$\tan\delta$ = The loss tangent (the loss tangent is a parameter which characterizes the electrical losses of a dielectric material[2]).

Another parameter of interest is the penetration depth (d) given by:

$$d = \frac{1}{\sqrt{2}\pi f \sqrt{\mu_o \varepsilon_o \varepsilon'_r \, (\sec\delta - 1)}} \qquad [2]$$

Here ε_o is the permittivity of free space, 8.854×10^{-4}F/cm; f is the frequency in hertz and μ_o is the permeability of free space, $4\pi \times 10^{-9}$H/cm.

By using the von Hippel method, information is obtained from which the dielectric properties of oil shale materials may be calculated. The voltage standing-wave ratio (VSWR) in front of a sample material, placed at the end of a coaxial or rectangular guide terminated in a short-circuit, is determined with the sample in place. The shift in the position of the voltage minimum due to the presence of the sample is also determined. This information, together with the sample dimensions, frequency and characteristics of the waveguide, permits determination of the dielectric properties[5].

The main achievement in this thesis has been to solve a complex transcendental equation on the computer by using a numerical analysis technique. It is solved by an interative technique using "Muller Quadratic Interpolation[6]".

Only those equations describing this method of measurement and those relationships necessary for the understanding of the computer program will be presented in this thesis.

CHAPTER 2

VON HIPPEL METHOD OF THE DIELECTRIC PROPERTIES OF OIL SHALE

The Von Hippel method allows us to obtain information from which the dielectric properties of oil shale materials may be calculated. This method was first introduced in 1946 and still remains one of the most commonly used.

In this method, the dielectric constant and loss tangent of either high- or low-loss materials are determined by placing a sample of the material, preferably $\lambda/4$ thick, in a shorted section of a waveguide and noting the change in the standing-wave pattern, and the snift of the voltage standing wave minimum resulting from the presence of the sample.

2.1 Measurement techniques

Figure 2 represents the short-circuited end of a rectangular waveguide containing a sample of length d in contact with the short circuit. The incident and reflected wave combine to form standing-waves in the waveguide. A voltage standing-wave pattern with and without the sample in the end of the waveguide is shown in fig. 3; where it may be noted that the wavelength in the sample, medium 2, is shorter than that in the empty waveguide, medium 1. Voltage minimas are spaced at half-wavelength intervals in the waveguide.

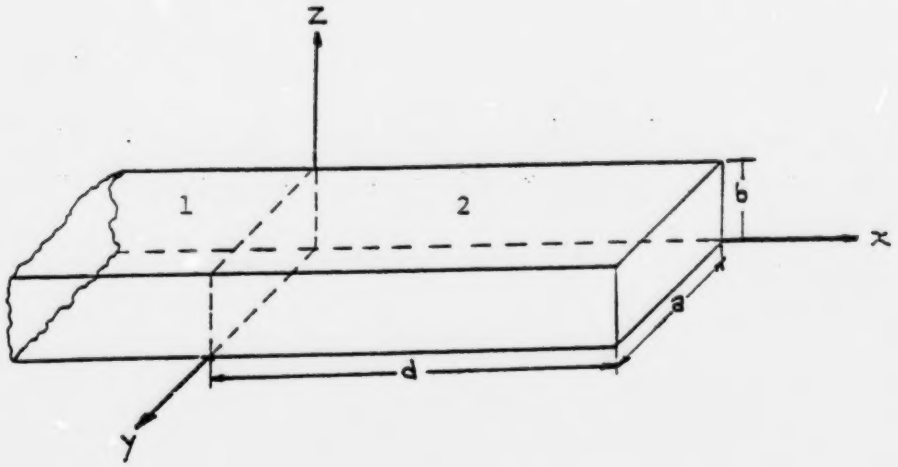


Figure 2 Short-circuited rectangular waveguide with a dielectric sample of length d at the shorted end.

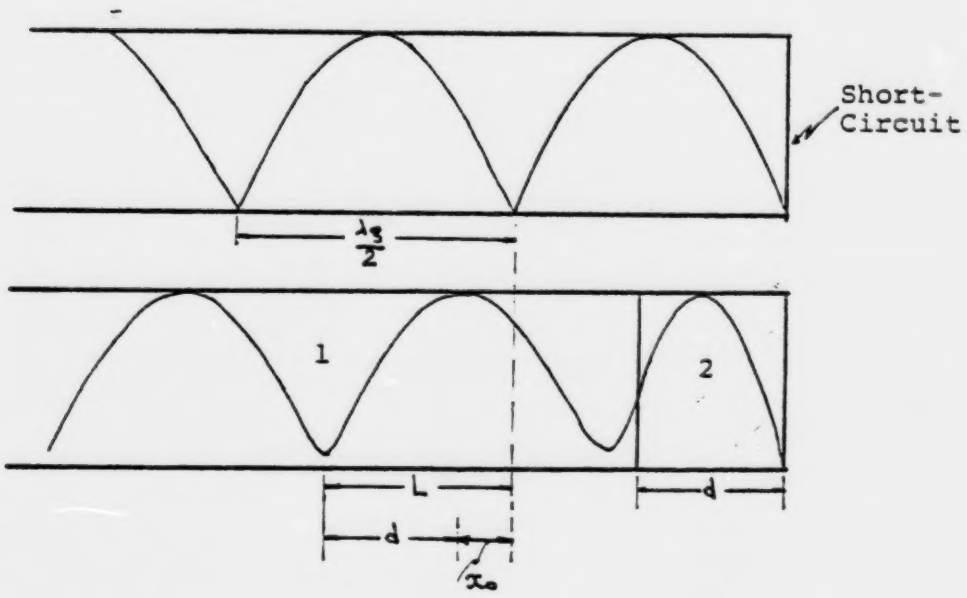


Figure 3 Voltage standing-wave pattern relationship in an empty waveguide and in a waveguide with the dielectric sample in place.

In Fig. 3, λ_g represents the guide wavelength in the empty waveguide. This guide wavelength is calculated from the frequency:

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - (f_c/f_o)^2}} = \frac{\lambda_o}{\sqrt{1 - (\lambda_c/\lambda_o)^2}} \qquad [3]$$

where λ_o is the free space wavelength corresponding to $f = c/\lambda_o$ (c = velocity of propagation in free space, 2.997925×10^{10} cm/sec.), and f_c and λ_c are the cut-off frequency and cut-off wavelength, respectively, for the waveguide ($f_c \lambda_c = c$).

The measurements of the relative levels of Emax to Emin, at two positions in the guide, suffices to determine the voltage standing-wave ratio (VSWR).

Also from Fig. 3, L represents the distance towards the generator from a short circuit reference plane to the first voltage minimum. The determination of x_o is found from this Fig. 3 to be

$$x_o = L - t \qquad [4]$$

2.2 Equations describing the method

The original equations presented by Von Hippel[1] are as follows:

$$\bar{\varepsilon_r} = \varepsilon_r' - j\varepsilon_r'' = [\frac{\lambda_o}{\lambda_c}]^2 - [\frac{\gamma_2 d}{2\pi d} \lambda_o]^2 \qquad [5]$$

and

$$\frac{\tanh\gamma_2 d}{\gamma_2 d} = -j\frac{\lambda_g}{2\pi d}\left[\frac{\frac{1}{VSWR} - j\tan\frac{2\pi x_o}{\lambda_g}}{1 - j\frac{1}{VSWR}\tan\frac{2\pi x_o}{\lambda_g}}\right] = Ce^{j\xi} \qquad [6]$$

(Note: A complete mathematical description of Eq. 6 is shown in appendix IV.)

where:

ε_r = The complex dielectric constant of the sample.

$\gamma_2 = \alpha_2 + j\beta_2$ = The complex propagation constant in the sample.

α = The attenuation constant.

β = The phase constant.

VSWR = Voltage standing-wave ratio as described in Section 2.1.

λ_g = Guide wavelength.

d = Sample thickness.

x_o = Distance from the sample interface to the first minimum voltage (Eq. 4).

Determination of the VSWR, λ_g, x_o and d, permits the right hand side of Eq. 6 to be reduced to a complex number, $Ce^{j\xi}$, and $\gamma_2 d$ may be calculated by using complex charts for hyperbolic functions or by the computer using numerical techniques.

2.3 Approximated solution to $\frac{\tanh\gamma_2 d}{\gamma_2 d} = Ce^{j\xi}$

If we consider a case when the dielectric is of low-loss then we may approximate $\gamma_2 d$ by $j\beta_2 d$ (attenuation is neglected). This in effect means that the VSWR is of such a high value that the inverse VSWR may be considered to be zero. With this approximation the transcendental equation (Eq. 6) takes on the following form[2]:

$$\frac{\tan\beta_2 d}{\beta_2 d} = -\frac{\lambda_g}{2\pi d} \tan\frac{2\pi x_o}{\lambda_g} \qquad [7]$$

The above equation involves no complex numbers and may be readily solved for $\beta_2 d$.

The following expression for $\alpha_2 d$ from the paper of Dakin and Works[4], and Eq. 7 are used as an approximation for $\gamma_2 d$.

$$\alpha_2 d = \left[\frac{\lambda_g \beta_2 d}{2\pi d}\right]\left[\frac{1}{VSWR}\right]\left[\frac{\tan\frac{2\pi x_o}{\lambda_g}}{1+\tan^2\beta_2 d+\frac{\tan\beta_2 d}{\beta_2 d}}\right] \qquad [8]$$

These last two equations, Eqs. 7 and 8, are considered later in the computer program as the first approximation to the true solution of Eq. 4.

CHAPTER 3

COMPUTER PROGRAM DESCRIPTION

This computer program was designed for general use and accomodates input data taken from measurements on coaxial lines and rectangular waveguides. It solves for either low-loss or high-loss dielectric samples and prints out the final values for ε_r', $\tan\delta$, d and checks the results against the original measurements. The regenerated measured data are also printed for comparison and detection of false roots.

The program was written to be used on an IBM 370 computer using FORTRAN IV programming language.

The program is available in the form of a subroutine called DIELEC.

In the calling sequence of DIELEC,

CALL DIELEC (A1,A2,L,THICK,FGI,CODE,ESTDC,TEMP,NSET),

A1,A2 = The readings from a calibrated attenuator; from them the program calculates the VSWR as follows:

$$VSWR = 10^{\frac{A1 - A2}{20}} \qquad [9]$$

L = The distance towards the generator from a short-circuit reference plane to the first voltage minimum (Fig. 3) (in cm)

FGI = Frequency in hertz

CODE = Sample identification (up to 12 characters)

ESTDC = Estimated dielectric constant, ε_r' (if no value for ε_r' is known, enter zero)

TEMP = Sample temperature in degrees centigrade

NSET = 1 only if heading table is required. If NSET equals any other integer no heading table is printed.

3.1 Subprogram DIELEC

With the input data in the computer and the calling routine being executed, calculations then proceed as follows.

1) Wavelength in air $\lambda_o = c/f$
2) Waveguide cut-off wavelength, $\lambda_c = 2a$ for rectangular guide or $\lambda_c = \infty$ for coaxial lines.
3) Guide wavelength (Eq. 3)

Next, the program calculates x_o according to Eq. 4.

In order to use Eq. 5 for computation the solution of Eq. 6 must be obtained. To solve Eq. 6 the right hand side must be rationalized and separated into its real and imaginary parts. This may be rewritten[5]:

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{\lambda_g \tan^2(S^2-1) - j\,\lambda_g S(1+\tan^2 U)}{2\pi d(1+S^2\tan^2 U)} \qquad [10]$$

where S represents the inverse standing-wave ratio, and U represents $\frac{2\pi x_o}{\lambda_g}$. For computer solutions, close estimates for the components of $\gamma_2 d = \alpha_2 d + j\beta_2 d$ are needed. For

given values of $\alpha_2 d$ and $\beta_2 d$, an expansion of $\frac{\tanh \gamma_2 d}{\gamma_2 d}$ is used to calculate real and imaginary parts in a subroutine called MULLER, that is to be described later.

An initial estimate of $\beta_2 d$ is obtained from Eq. 7 (for low-loss cases; $\tan\delta < 0.1$) by solving for $\beta_2 d$ using the estimated value from the data input for ε_r'. In the program, if a zero value is entered in the calling sequence for the estimated dielectric constant, a default value of 2.5 is assigned. Provision is also made in the program to use the dielectric constant calculated for the previous set of data as the estimated dielectric constant for the next set of measurements in a similar sample. This is valid and efficient due to the gradual changes observed in the dielectric properties across the band of frequency studied.

In the lossless case $\alpha_2 = 0 = \zeta$, $\frac{\tanh \gamma_2 d}{\gamma_2 d} = Ce^{j\xi}$ becomes, in accordance with Eq. 6

$$\frac{\tan \beta_2 d}{\beta_2 d} = C \qquad [11]$$

C is calculated as the magnitude of the right-hand side of Eq. 6 because for lossy dielectric samples, C will include a contribution due to the loss and will provide a better estimate than might be obtained using Eq. 7[5].

The estimate $\beta_2 d$ and a value of $\alpha_2 d$, calculated from Eq. 8 using this improved estimate of $\beta_2 d$, then comprise the first estimate of $\gamma_2 d$.

Since $\frac{\tanh \gamma_2 d}{\gamma_2 d}$ is a multivalued function and each solution to Eq. 6 results in a different set of values for ε_r' and ε_r'', this program only takes the first set of $\alpha_2 d$ and $\beta_2 d$, satisfying Eq. 10 from which to proceed with the calculations. This is done because subsequent roots to Eq. 6 gives unreasonable solutions for ε_r' and ε_r''. The relative permittivity or dielectric constant is then calculated from Eq. 5.

The associated loss tangent for this set of values for ε_r' and ε_r'' is calculated ($\tan\delta = -\frac{\varepsilon_r''}{\varepsilon_r'}$).

At the end of the calculations for a given set of data, the program prints out a summary table including sample identification, frequency, measured VSWR, measured dmin (x_o), sample thickness, dielectric constant, loss tangent, penetration depth and checks values calculated to compare against the original measurements. Examples of input card data for rectangular and coaxial lines are shown in Appendix I. The summary tables are shown in Appendix III.

3.2 Subroutines used by DIELEC

SUBROUTINE BETA2D:

This subroutine is used to improve the estimate of $\beta_2 d$ and has been taken out of reference 3 without modification.

In the calling sequence of BETA2D:

CALL BETA2D (C, BETA)

C = The magnitude of the right hand side of Eq. 6, with proper sign

BETA = On input, BETA is an approximation to the solution of $\frac{\tan \beta_2 d}{\beta_2 d} = C$, obtained from measurement data and the estimate of ε_r' from input data. On output, BETA contains the exact root to Eq. 11 found by BETA2D .

SUBROUTINE MULLER:

Following the use of the subroutine BETA2D to improve the estimate of $\beta_2 d$ and the subsequent calculations of an estimate for $\alpha_2 d$, the MULLER subroutine is used to adjust the values of the components of $\gamma_2 d = \alpha_2 d + j\beta_2 d$ to satisfy Eq. 6.

In the calling sequence of MULLER:

CALL MULLER (X, GAMMAD)

X = A complex vector. It is the first approximation to the solution of Eq. 6, obtained from the solution of Eq. 11 and from the calculated value for $\alpha_2 d$ from Eq. 8

GAMMAD = On output, GAMMAD contains the exact root to Eq. 6 found by MULLER

The algorithm of this subroutine is based on the Muller Quadratic interpolation. This method is used to find the zeros of Eq. 6. The method is iterative and converges

almost quadratically in the vicinity of a root. Its convergence is based on the following criteria:

$$\left| \frac{x_i - x_{i-1}}{x_i} \right| < \varepsilon$$

x_i = The new root found

x_{i-1} = The previous root found

ε = A prescribed tolerance

A complete description of this method can be obtained from 'Elementary Numerical Analysis: an algorithmic approach', S.D. Conte, pp 67-68.

3.3 Overall flowchart of the main program with example of calling sequence

Having briefly described an interrelation of the subprograms, we shall now look at the overall organization of the main program in the form of a flowchart as shown in Fig. 5.

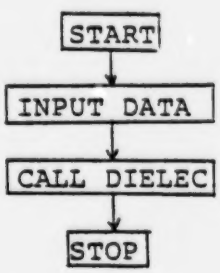


Figure 5. Overall Organization of the Main Program

Let us now look at a complete calling sequence with the main program.

```
      Read, NCODE
NCODE:  number of sample materials to be processed
      DO 10 I = 1,NCODE
      Read, CODE,NFREQ,ESTDC
      DO 20 NSET = 1,NFREQ
      Read, A1,A2,L,THICK,FGI,TEMP
      CALL DIELEC (A1,A2,L,THICK,FGI,CODE,ESTDC,TEMP,NSET)
   20 CONTINUE
   10 CONTINUE
      RETURN
      END
```

Input should be submitted as follows.

First Card: NCODE

Second Card: CODE,NFREQ,ESTDC

Third Card: A1,A2,L,THICK,FGI,TEMP

Output would be given as those shown in Appendix III.

CHAPTER 4

CONCLUSIONS AND RESULTS

The main contributions of this work are:

1) One of the guidelines used in writing the program was to make the algorithm easy to understand and simple to use.

2) The program exists in the form of a subroutine called DIELEC and can be stored in the FORTRAN library for general use.

3) The program is capable of generating its own initial approximation to the solution, thereby eliminating the need for the user to supply the initial guess. The algorithm-generated approximation facilitates execution of the program.

4) DIELEC executes in double precision; however, accuracy of results is limited by the accuracy of the measurement of x_o and VSWR.

5) Calculations of ε_r' based on measurement made on samples of teflon and plexiglass were compared to published values[1] with a maximum percentage difference equal to 0.98 (see table I). A maximum of 17 iterations were required.

TABLE I

MATERIAL	CALCULATED ε_r'	ε_r' FROM TABLES	% DIFFERENCE
TEFLON	2.01	2.03	0.98
PLEXIGLASS	2.01	2.59	0.77

REFERENCES

1. Arthur von Hippel, "Dielectric and Waves", John Wiley and Sons, Inc., New York.

2. G.A. Hickey, "Dielectric Measurements", Undergraduate Thesis, Electrical Engineering Department, UNB.

3. S. Roberts and A. von Hippel, "A New Method for Measuring Dielectric Constant and Loss Tangent in the Range of Centimeters Wave ", J. Appl. Phys., 17(7): 610-616.

4. T.W. Dakin and C.N. Works, "Microwave Dielectric Measurements", J. App. Phys., 18(9): 789-796.

5. S.O. Nelson, C.W. Schlaphoff and L.E. Stetson, "A Computer Program for Short-Circuited Waveguide Dielectric Properties Measurements on High- or Low-loss Materials", The Journal of MIcrowave Power, Vol. 8, No. 1, March 1973: 13-22.

6. S.D. Conte, "Elementary Numerical Analysis: an Algorithm Approach", pp 67-68.

APPENDIX I

PROGRAM LISTING

```
                                                
```

```
      DMIN=DMIN1-THICK
5     IF(DMIN.GE.0.0) GO TO 6
      DMIN=DMIN+WGUIDE/2.0
C I.E ADD HALF-WAVELENGHTS UNTIL DMIN BECOMES POSITIVE
      GO TO 5
6     U=2.0*PI*DMIN/WGUIDE
      T=TAN(U)
      P=(WFREE/WCOFF)**2
      P1=2.0*PI/WFREE
      Q=WGUIDE/(2*PI*THICK)
      S1=WFREE/(2*PI*THICK)
      DEN=Q/(1+(S*T)**2)
      A=T*(S**2-1)
      B=S*(1+T**2)
      C=SQRT(A*A+B*B)*DEN
      C1=C*SIGN(1.0,A)
      U0=A*DEN
      V0=-B*DEN
      RHS=CMPLX(U0,V0)
C BETA IS THE FIRST ESTIMATE OF B2D,COMPUTED FROM E'
      IF(ESTDC.EQ.0.0) ESTDC=ESTEPM
      BETA=SQRT(ESTDC-P)/S
C    QUAD CORRESPONDS TO THE QUADRANT INTO WHICH BETA (IN RADIANS) FALLS
      QUAD=1
      WHILE(BETA.GT.PI)DO
      BETA=BETA-PI
      END WHILE
8     IF(BETA.LT.QUAD*PI/2) GO TO 9
      QUAD=QUAD+1
      GO TO 8
C SOLUTIONS DO NOT EXIST IN THE FIRST QUADRANT IF C.LT.1
9     IF(QUAD-1) 7,7,40
7     IF(C1.LE.1) BETA=BETA+PI
40    BETA=BETA
      CALL BETA2D(C1,BETA)
      BETA=BETA
      T1=TAN(BETA)
      ALPHA=BETA*Q*S*(1.0+T**2)/((1.0+T1**2)-T1/BETA)
      X(1)=CMPLX(ALPHA,BETA)
      IF(CABS(APPX).NE.0.0) X(1)=APPX
      CALL MULLER(X,GAMMAD,N)
      DO 100 I=1,N
      APPX=GAMMAD(I)
      CDC=P-(GAMMAD(I)*S1)**2
      E2=REAL(CDC)
      E22=AIMAG(CDC)
      RP=E2
      ESTDC=RP
      TD=-E22/E2
      LANDA=DATAN(TD)
      SECG=1.0/DCOS(LANDA)
      E0H0RP=DSQRT((E0*H0*RP)/2.0)
      SECG1=DSQRT(SECG-1.0)
      ALFA=2.0*PI*FG*E0H0RP*SECG1
      D=1.0/ALFA
      KC=(2.0*PI/WCOFF)**2
      K0=((2.0*PI*FG)**2)*E0*H0
      KCK0=KC-K0
      RESQ=KC-RP*K0
      IMSQ=RP*TD*K0
```

```
      CPLS=CMPLX(RESQ,IMSQ)
      CPLSQ=CSQRT(KCKQ/CPLS)
      GAM1=CSQRT(CPLS)
      GAML1=GAM1*THICK
      EXPQ=CEXP(-2.0*GAML1)
      TAHYT=1.0-EXPQ
      TAHYB=1.0+EXPQ
      CALL TRAPS(1.1,999.1.1)
      TAHY=TAHYT/TAHYB
      TOP12=CPLSQ*TAHY
      BOT12=TOP12
      ROIT=TOP12-1.0
      ROIB=BOT12+1.0
      ROI=ROIT/ROIB
      AROIR=REAL(ROI)
      AROII=AIMAG(ROI)
      RO=CABS(ROI)
      CVSWR=(1.0+RO)/(1.0-RO)
      AROI=ATAN2(AROII,AROIR)
      CDMIN=(WGUIDE/4.0)*((AROI/PI)+1.)
      PRINT 2,FGI,TEMP,VSWR,CVSWR,DMIN,CDMIN,THICK,RP,TD,Q
    2 FORMAT(3X,F6.3,4X,F6.2,4X,F6.3,5X,F6.3,5X,F6.3,5X,F6.3,6X,F6.4,5X,
     *F6.2,4X,F6.4,5X,F6.2/)
  100 CONTINUE
   55 CONTINUE
   50 CONTINUE
      RETURN
      END

      COMPLEX FUNCTION FCT(Z)
      COMPLEX Z,RHS
      COMPLEX TOP,BOT
      COMMON RHS
      FCT=CEXP(-2.0*Z)-(1.0-Z*RHS)/(1.0+Z*RHS)
      RETURN
      END
C     ************************************
C     *     ""SUBROUTINE BETA2D""        *
C     *    SUBROUTINE BETA2D IS USED     *
C     *    TO SOLVE TAN(B2D)/(B2D)=C     *
C     *    FOR B2D WHEN C IS KNOWN       *
C     *    FROM MEASUREMENT DATA         *
C     ************************************

      SUBROUTINE BETA2D(C,BETA)
      K=1
      PI=3.141592654
      X0=BETA
      NSTOP=0
   10 IF(BETA.LT.K*PI/2) GO TO 20
      K=K+1
      GO TO 10
   20 TEST=TAN(BETA)/BETA
      IF(C*TEST.GT.0.0) GO TO 21
      IF(ABS(C).GT.2.0) GO TO 30
   21 DO 26 N=1,20
      IF(K-(2*N-1)) 25,22,26
   22 IF(K-1) 23,23,24
   23 A=0
      B=PI/2
```

```
      GO TO 40
24    B=K*PI/2
      A=B-PI
      GO TO 40
25    A=(K-1)*PI/2
      B=A+PI
      GO TO 40
26    CONTINUE
30    IF(C) 31,32,32
31    BETA=K*PI-BETA
      A=K*PI/2
      B=A+PI
      GO TO 40
32    BETA=(K-1)*PI-BETA
      B=(K-1)*PI/2
      IF(K-2) 33,33,34
33    A=0
      GO TO 40
34    A=B-PI
40    X1=BETA
50    DELTA=X1*(TAN(X1)-C*X1)/(X1/COS(X1)**2-TAN(X1))
      X2=X1-DELTA
      NSTOP=NSTOP+1
      IF(X2.LT.A) GO TO 60
      IF(X2.GE.B) GO TO 60
      IF(ABS(DELTA).LE.0.000001) GO TO 80
      X1=X2
      GO TO 70
60    IF(C.GT.0) BETA=(BETA+B)/2
      IF(C.LE.0) BETA=(BETA+A)/2
      X1=BETA
70    IF(NSTOP.EQ.100) GO TO 90
      GO TO 50
80    BETA=X2
      RETURN
90    BETA=XJ
      RETURN
      END
C     ************************************
C     *      ""SUBROUTINE MULLER""       *
C     * SUBROUTINE MULLER IS USED TO     *
C     * SOLVE THE TRANSCENDENTAL EQ.     *
C     *          tanh γ2d                *
C     *          ------- = ce^jξ         *
C     *            γ2d                   *
C     *          SEGUNDO                 *
C     *           VASQUEZ                *
C     ************************************
      SUBROUTINE MULLER(X,GAMMAD,N)
      COMPLEX XT,H,F1,F2,F3,LAMBDA,DELTA,G,FPRT,TEMP,RT(16)
      COMPLEX Y,CSQRT,CMPLX,D,D1,D2,XN,FCT
      COMPLEX X(100),GAMMAD(100)
      EPS=1.0E-7
      K=0
100   K=K+1
      RT(K)=0.0
      XT=0.9*X(K)
      H=-0.1*X(K)
      M=3
```

```
      GO TO 11
    3 F1=FPRT
      XT=1.1*X(K)
      M=4
      GO TO 11
    4 F2=FPRT
      XT=X(K)
      M=5
      GO TO 11
    5 F3=FPRT
      LAMBDA=CMPLX(-0.5,0.0)
   30 DELTA=LAMBDA+1
      G=F1*LAMBDA**2-F2*DELTA**2+F3*(LAMBDA+DELTA)
      D=G*G-4.0*F3*LAMBDA*DELTA*(F1*LAMBDA-F2*DELTA+F3)
      Y=CSQRT(D)
      D1=G+Y
      D2=G-Y
      IF(CABS(D1)-CABS(D2))200,200,201
  200 D=D2
      GO TO 21
  201 D=D1
   21 CONTINUE
      LAMBDA=-(2.0*F3*DELTA)/D
      H=LAMBDA*H
      XN=XT+H
      IF(ABS(REAL(XN-XT)).LE.EPS.AND.ABS(AIMAG(XN-XT)).LE.EPS)GO TO 22
      XT=XN
      M=6
      GO TO 11
   22 GAMMAD(K)=XN
      IF(K.EQ.N) GO TO 23
      GO TO 100
   11 FPRT=FCT(XT)
      IF(K.EQ.1) GO TO 14
      J=1
   13 J=J+1
      TEMP=XT-RT(J-1)
      FPRT=FPRT/TEMP
      IF(J.EQ.K) GO TO 14
      GO TO 13
   14 L=M-2
      GO TO(3,4,5,6),L
    6 F1=F2
      F2=F3
      F3=FPRT
      GO TO 30
   23 RETURN
      END

$ENTRY
```

APPENDIX II

MATHEMATICAL THEORY

A standing wave pattern in a waveguide can be represented as the sum of two travelling waves progressing in opposite directions parallel to the axis of symmetry (see Fig. 1). The transverse field components E(x) and H(x) in air (medium 1) at a variable distance x from the dielectric surface along a line parallel to the axis are given below[4].

$$E_1(x) = Ai_1 e^{\gamma_1 x} + Ar_1 e^{-\gamma_1 x}$$

$$= Ai_1 (e^{\gamma_1 x} + \rho_T e^{-\gamma_1 x})$$

$$H_1(x) = \frac{Ai_1}{Z_1} e^{\gamma_1 x} - \frac{Ar_1}{Z_1} e^{-\gamma_1 x}$$

$$= \frac{Ai_1}{Z_1} (e^{\gamma_1 x} - \rho_T e^{-\gamma_1 x}),$$

Ai_1 and Ar_1 are the amplitude of the incident and reflected waves at the dielectric surface. Their ratio

$$\frac{Ar_1}{Ai_1} = \rho_T \triangleq e^{-2\phi}$$

where

$$\phi = P + jQ$$

defines a reflection coefficient which characterizes the terminating waveguide section containing the dielectric medium 2.

The ratio of the two field vectors at the boundary (x=0), (Fig. 1), is related to ρ_T and ϕ.

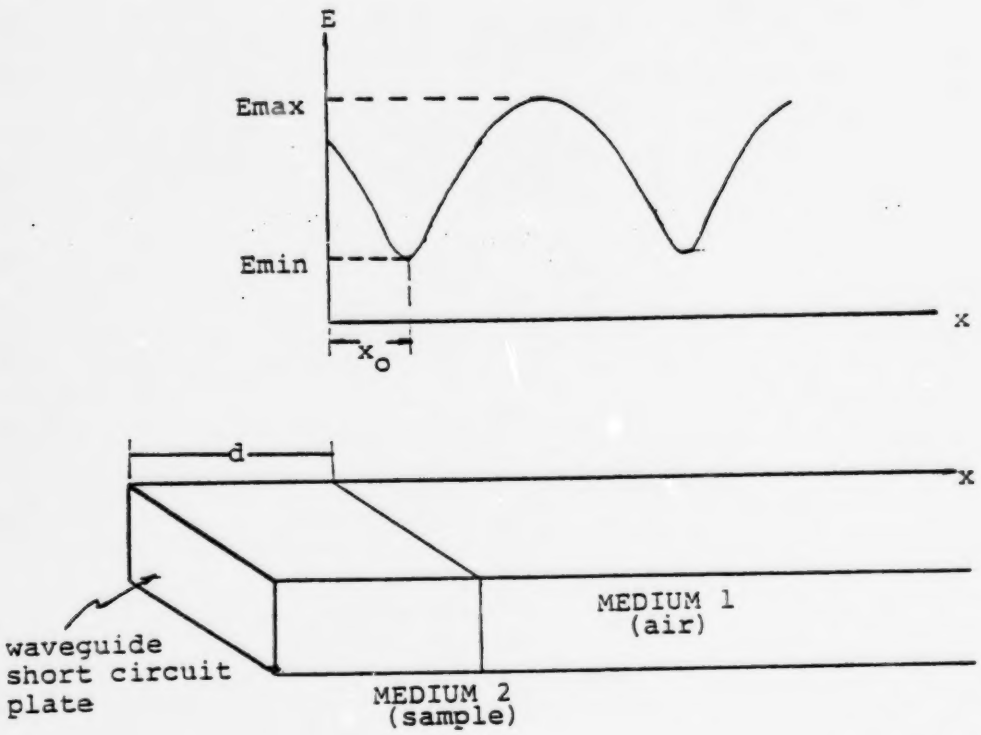


Figure 1. Diagram showing waveguide and sample geometry

$$Z(o) = \frac{E_1(o)}{H_1(o)} = Z_1 \frac{1+\rho_T}{1-\rho_T} = Z_1 \frac{1+e^{-2\phi}}{1-e^{-2\phi}} = Z_1 \coth\phi \quad [4]$$

This is known as the terminating impedance of medium 1.

Normally the attenuation in the air-filled part of the waveguide can be neglected, that is, the propagation γ_1, is reduced to its phase factor

$$\gamma_1 = \alpha_1 + j\beta_1 = j\beta_1 = j\frac{2\pi}{\lambda_g}$$

where λ_g is the wavelength in air inside the waveguide. Since γ_1 is imaginary, it follows from (1) that the maximum

and minimum field amplitude of the standing wave are:

$$Emax = |Ai|(1 + |\rho_T|) = |Ai|(1 + e^{-2P}) \qquad [5]$$

$$Emin = |Ai|(1 - |\rho_T|) = |Ai|(1 - e^{-2P})$$

and their ratio is

$$\frac{Emax}{Emin} = \frac{1 + e^{-2P}}{1 - e^{-2P}} = \frac{1}{\tanh P}$$

$$= \frac{1 + |\rho_T|}{1 - |\rho_T|} \qquad [6]$$

This ratio of maximum to minimum electric field intensity is called the voltage standing-wave ratio (VSWR).

The first minimum of the E-wave from the boundary of the dielectric is reached at a distance x_o where the incident and reflected waves have a phase difference of π radians, that is[2],

$$2Q = 4\pi(\frac{1}{4} - \frac{x_o}{\lambda_g}) \qquad [7]$$

The terminating impedance Z(o), or the impedance at the interface, can be formulated in terms of the directly measurable parameters VSWR, x_o, and λ_g. By expanding cothϕ and recalling the relation between Q and x_o (eq. 7) we obtain the desired expression.

$$Z(o) = Z_1 \frac{\tanh P - j \cot Q}{1 - j \tanh P \cot Q}$$

$$= Z_1 \frac{\frac{1}{VSWR} - j \tan \frac{2\pi x_o}{\lambda_g}}{1 - j \frac{1}{VSWR} \tan \frac{2\pi x_o}{\lambda_g}} \qquad [8]$$

The standing wave in medium 2 is

$$E_2(x) = Ai_2 e^{\gamma_2 x} + Ar_2 e^{-\gamma_2 x}$$
$$H_2(x) = \frac{Ai_2}{Z_2} e^{\gamma_2 x} - \frac{Ar_2}{Z_2} e^{-\gamma_2 x} \qquad [9]$$

and at x = -d (Fig. 1) the waveguide is terminated by a metal place introducing at this boundary a voltage node

$$0 = Ai_2 e^{-\gamma_2 d} + Ar_2 e^{\gamma_2 d} \qquad [10]$$

Solving [10] for Ar_2/Ai_2, and substituting this value in [9] gives the terminating impedance for this short-circuit measurement of medium 2

$$Z(o) = \frac{E_2(o)}{H_2(o)} = Z_2 \tanh \gamma_2 d \qquad [11]$$

The characteristic impedance Z_2 and the propagation function γ_2 of any TE mode in a dielectric are related in the following manner:

$$\gamma_2 Z_2 = j \omega \mu_2,$$

where μ_2 is the premeability of the dielectric medium. The permeability of dielectric materials (non-magnetic) is generally μ_o of free space. Therefore

$$\gamma_2 Z_2 = j \omega \mu_o = \gamma_1 Z_1$$

and

$$Z(o) = Z_1 \frac{\gamma_1}{\gamma_2} \tanh \gamma_2 d \qquad [12]$$

Equating [12] and [8] yields

$$\frac{\tanh\gamma_2 d}{\gamma_2 d} = -j\frac{\lambda_g}{2\pi d}\left[\frac{\frac{1}{VSWR} - j\tan\frac{2\pi x_o}{\lambda_g}}{1 - j\frac{1}{VSWR}\tan\frac{2\pi x_o}{\lambda_g}}\right] = Ce^{j\xi} \qquad [13]$$

All the terms in the right hand side of the above equation are known or measurable so that C and ξ are known. Now this is a transcendental equation which could be solved by the use of complex function charts or on the computer using numerical analysis technique to give γ_2.

The relationship between ε_r and γ_2 depends on the cut-off wavelength of the particular waveguide. The cut-off wavelength λ_c is the longest free-space wavelength for which propagation in a particular mode is possible. It may be expressed in terms of the cross-section dimensions of the waveguide: for a rectangular waveguide (and a TE_{10} mode) $\lambda_c = 2a$; where a is the width of the guide; and for coaxial line, or free space (tem-mode) $\lambda_c = \infty$. For TE-modes in hollow waveguide[2]

$$\varepsilon_r = \varepsilon_r' - j\,\varepsilon_r'' = \left(\frac{\lambda_o}{\lambda_c}\right)^2 - \left(\frac{\gamma_2 d}{2\pi d}\,\lambda_o\right)^2 \qquad [14]$$

In free space or coaxial line this equation reduces to

$$\varepsilon_r = \varepsilon_r' - j\,\varepsilon_r'' = \left(\frac{\gamma_2 d}{2\pi d}\,\lambda_g\right)^2, \qquad [15]$$

since λ_g has been measured or can be determined once frequency is known and λ_c is known, ε_r can be determined.

APPENDIX III

SAMPLE OUTPUT

DIELECTRIC PROPERTIES OF OIL SHALE

SAMPLE NO. AX A10-03P

FREQUENCY GHZ	TEMP CELSIUS	VSWR MEASURED	VSWR CALCULATED	DMIN MEASURED CM	DMIN CALCULATED CM	SAMPLE THICKNESS CM	DIELECTRIC CONSTANT	LOSS TANGENT	PENETRATION DEPTH CM
8.200	50.00	17.783	17.782	3.012	3.012	1.0150	4.09	0.0943	6.11
8.200	100.00	12.303	12.302	2.882	2.882	1.0150	5.14	0.1190	4.32
8.200	150.00	10.351	10.350	2.772	2.772	1.0150	5.93	0.0898	5.33
8.200	200.00	15.311	15.310	3.002	3.002	1.0150	4.16	0.1109	5.16
8.200	250.00	19.724	19.724	3.002	3.002	1.0150	4.17	0.0853	6.69
8.200	300.00	27.861	27.860	3.002	3.002	1.0150	4.18	0.0600	9.49
9.000	50.00	13.646	13.645	2.228	2.228	1.0150	4.51	0.0752	6.64
9.000	100.00	21.014	21.013	2.288	2.288	1.0150	4.12	0.0584	8.96
9.000	150.00	8.222	8.222	2.208	2.208	1.0150	4.68	0.1157	4.25
9.000	200.00	11.350	11.349	2.268	2.268	1.0150	4.24	0.1040	4.94
9.000	250.00	17.783	17.782	2.288	2.288	1.0150	4.12	0.0692	7.56
9.000	300.00	24.266	24.265	2.298	2.298	1.0150	4.05	0.0516	10.22
0.000	50.00	6.761	6.760	1.691	1.691	1.0150	4.10	0.0885	5.33
0.000	100.00	4.732	4.731	1.681	1.681	1.0150	4.20	0.1185	3.94
0.000	150.00	4.074	4.073	1.661	1.661	1.0150	4.30	0.1242	3.71
0.000	200.00	5.158	5.158	1.661	1.661	1.0150	4.25	0.1008	4.60
0.000	250.00	10.000	9.999	1.681	1.681	1.0150	4.11	0.0579	8.13
0.000	300.00	13.183	13.181	1.691	1.691	1.0150	4.06	0.0459	10.31
1.000	50.00	2.113	2.113	1.083	1.083	1.0150	4.17	0.0927	4.58
1.000	100.00	1.496	1.496	1.033	1.033	1.0150	4.29	0.1183	3.55
1.000	150.00	1.413	1.412	1.053	1.053	1.0150	4.29	0.1265	3.32
1.000	200.00	1.585	1.585	1.023	1.023	1.0150	4.28	0.1114	3.77
1.000	250.00	3.162	3.162	1.103	1.103	1.0150	4.10	0.0659	6.50
1.000	300.00	4.097	4.097	1.113	1.113	1.0150	4.07	0.0524	8.21
2.000	50.00	3.255	3.255	0.487	0.487	1.0150	4.05	0.0600	6.58
2.000	100.00	2.661	2.661	0.395	0.395	1.0150	4.17	0.0847	4.60
2.000	150.00	2.585	2.586	0.395	0.395	1.0150	4.17	0.0870	4.48
2.000	200.00	2.661	2.661	0.395	0.395	1.0150	4.17	0.0847	4.60
2.000	250.00	5.309	5.309	0.502	0.502	1.0150	4.05	0.0362	10.92
2.000	300.00	5.957	5.957	0.502	0.502	1.0150	4.05	0.0322	12.25